CHAPTER-1 - RELATIONS AND FUNCTIONS

Gist / Summary of the lesson:

- **Types Of Relations:**
  Empty relation, Universal relation, Reflexive, Symmetric, Transitive and Equivalence relations.
- Types Of Functions: One – one (or injective) functions, onto (or surjective) functions, One-one and onto (or bijective)

Definitions:

- A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \emptyset \subset A \times A$
- A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$
- A relation R in a set A is called
  (i) reflexive, if $(a, a) \in R$, for every $a \in A$,
  (ii) symmetric, if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R$, for all $a_1, a_2 \in A$.
  (iii) transitive, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.
- A relation R in a set A is said to be an equivalence relation iff R is reflexive, symmetric and transitive
- A function $f : X \rightarrow Y$ is defined to be one-one (or injective), if the images of distinct elements of X under f are distinct, i.e., for every $x_1, x_2 \in X$, $x_1 \neq x_2$. implies $f(x_1) \neq f(x_2)$. Otherwise, f is called many-one.
- A function $f : X \rightarrow Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under f, i.e., for every $y \in Y$, there exists an element x in X such that $f(x) = y$. In other words $f: X \rightarrow Y$ is onto if and only if Range of f = Y.
- A function $f : X \rightarrow Y$ is said to be one-one and onto (or bijective), if f is both one-one and onto.

Formulae:

- If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and number of relations from set A to set B=2^{pq}.
- If A is a non-empty finite set containing n elements, then number of reflexive relations on set A=$2^{n(n-1)}$.
- If A is a non-empty finite set containing n elements, then number of symmetric relations on set A =$2^{\frac{n(n+1)}{2}}$.
- If A and B are non-empty finite sets containing m and n elements respectively, then
  (i) Number of functions from A to B = n^m
  (ii) Number of one-one functions from A to B =$\begin{cases} nP_m, & if\ m \leq n \\ 0, & if\ m > n \end{cases}$
  (iii)Number of onto functions from A to B
  $=\begin{cases} nC_0 n^m - nC_1 (n-1)^m + nC_2 (n-2)^m - \ldots\ldots\ldots, & if\ n \leq m \\ 0, & if\ n > m \end{cases}$

  (iv) Number of one-one and onto functions

  i.e., bijective functions from A to B= $\{ m!, if\ m = n$

MULTIPLE CHOICE QUESTIONS

1) Let A={3,5}. Then number of reflexive relations on A is

(A) 2 (B) 4 (C) 0 (D) 8

Solution: If A is a non-empty finite set containing n elements, then number of reflexive relations on set A=$2^{n(n-1)}$.

Here n=2

No. of reflexive relations on A= $2^{2(2-1)}=2^2=4$ **Answer: B**

2) The number of possible symmetric relations on a set consisting of 4 elements is

(A) 512 (B)1024 (C)256 (D)32

Solution: If A is a non-empty finite set containing n elements, then number of symmetric relations on set A $=2^{\frac{n(n+1)}{2}}$

Here $n = 4$

No. of symmetric relations on A=$2^{\frac{4(4+1)}{2}}=2^{\frac{4\times5}{2}}=2^{10}=1024$ **Answer: B**

3) A relation R on set G = $\{All\ the\ students\ in\ a\ certain\ mathematics\ class\}$ is defined as, $R = \{(x,y): x\ and\ y\ have\ the\ same\ mathematics\ teacher\}$.Which of the following is true about R ?

(A) R is reflexive and transitive but not symmetric.

(B)R is transitive and symmetric but not reflexive.

(C) R is reflexive and symmetric but not transitive.

(D) R is an equivalence relation.

Answer: D

4) Let the relation R in the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by $R = \{(a,b): |a-b|\ is\ a\ multiple\ of\ 4\}$. Then the equivalence class [1] is

(A)$\{1,5,9\}$ (B)$\{0,1,2,5\}$ (C)$\{1\}$ (D)A

Solution: Given $A = \{0,1,2,3,4,\dots,12\}$

Now $[1] = \{x \in A: |x-1|\ is\ a\ multiple\ of\ 4\} = \{1,5,9\}$ **Answer: A**

5) A and B are two sets with m elements and n elements respectively(m < n). How many onto functions can be defined from set A to set B?

(A) 0 (B)m! (C) n! (D)n^m

Solution: Given $n(A) = m, n(B) = n$

We know that onto function requires every element of set B to be mapped by at least one element from set A. Since it is given that m < n, there is no such onto function. Therefore number of onto functions from set A to set B where m < n is zero. **Answer: A**

6) For real x, let $f(x) = x^3. Then$

(A) f is one-one but not onto on R

(B) f is onto on R but not one-one.

(c) f is one-one and onto on R

(D) f is neither one-one nor onto on R

Solution: Let $f(x_1) = f(x_2)\ \forall\ x_1, x_2 \in R$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$So\ f(x)\ is\ one-one.$

$Let f(x) = x^3 = y$

$x = y^{\frac{1}{3}}, \forall y \in R$

Every image $y \in R$ has a unique pre image in R.

f is onto.

f is one-one and onto.

Answer: C

7) The function $f : R \rightarrow [-1,1]$ *defined by* $f(x) = \cos x$ is :

(A) Both one-one and onto (B) not one-one but onto

(C) One-one but not onto (D) neither one-one nor onto

Answer: B

8) A function f: R→ A defined as $f(x)=x^2+1$ is onto, if A is

(A) $(-\infty, \infty)$ (B) $(1,\infty)$ (C) $[1,\infty)$ (D) $[-1,\infty)$

Solution: $x \in R \Rightarrow x^2 \geq 0 \Rightarrow x^2+1 \geq 0+1 \Rightarrow f(x) \geq 1$

Range of f = $[1,\infty)$

Thus for f to be onto , A=$[1,\infty)$ **Answer: C**

9) Let L denotes the set of all straight lines in a plane. Let a relation R be defined by lRm if and only if l is perpendicular to m ∀ l, m ∈ L. Then R is:

(A) reflexive (B) symmetric (C) transitive (D) Equivalence relation

Answer: B

10) A relation R on set A=$\{x: x \in Z \text{ and } 0 \leq x \leq 10\}$ as R=$\{(x,y): x=y\}$ is given to be an equivalence relation. The number of equivalence classes is

(A)1 (B)2 (C)10 (D)11

Solution: A={0,1,2,3,4,5,6,7,8,9,10}

R={(0,0),(1,1), (2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8),(9,9),(10,10)}

We observe that each element in set A is only related to itself in relation R.

[0]={0}, [1]={1}, [2]={2}, [3]={3}, [4]={4}, [5]={5}, [6] ={6}, [7]={7}, [8]={8}, [9]={9}, [10]={10}

Number of equivalence classes is 11 **Answer: D**

ASSERTION AND REASON BASED QUESTIONS

Questions numbers 1 to 10 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

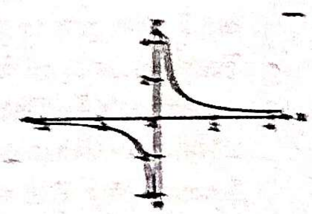
1) X = {0, 2, 4, 6, 8}. P is a relation on X defined by P = {(0, 2), (4, 2), (4, 6), (8, 6), (2, 4), (0, 4)}.

Assertion (A): The relation P on set X is a transitive relation.

Reason (R): The relation P has a subset of the form {(a, b), (b, c), (a, c)}, where a, b, c ∈ X is transitive.

Solution: P is not transitive as (4,2) (2,4)∈P but (4,4) ∉ P. **Answer: D**

Shown below is the graph of the function $f(x) = \frac{9-x^2}{9x-x^3}$



Assertion (A): The function f is not onto.

Reason (R): 3 ∈ R (co-domain of f) has no pre-image in the domain of f.

Solution: A is true but R is false. 0∈ R *does not* have a pre-image. **Answer: C**

2) Assertion (A): Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f+g)x = e^x + \log x$ where domain of $(f+g)$ is R.

Reason (R) : $Dom(f+g) = Dom(f) \cap Dom(g)$

Solution: Domain of $f = R$.

Domain of $g = (0, \infty)$

Domain of $(f+g) = R \cap (0,\infty) = (0,\infty)$ not R

Here Assertion is false, Reason is true. **Answer: D**

3) Assertion (A): Let A=$\{x \in R : -1 \le x \le 1\}$. If $f: A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R): If $y = -1 \in A$, then $x = \pm\sqrt{-1}$ not an element of A.

Solution: Here assertion and reason are true and R is the correct explanation of A.

Answer: A

4) Assertion (A): Let Z be the set of integers. A function f: Z→ Z defined as $f(x) = 3x - 5, \forall x \in Z$ is a bijective.

Reason (R): A function is bijective if it is both surjective and injective.

Solution: Here $f(x) = 3x - 5$ is not a bijective function. **Answer :D**

5) $f: X \to X$ is a function on the finite set X.

Assertion (A): If f is onto, then f is one-one and if f is one-one, then f is onto..

Reason (R): Every one-one function is always onto and every onto function is always one-one.

Solution: For function on finite sets, a one-one function implies onto and vice-versa. This is not necessarily true for infinite sets. **Answer: C**

So reason is not true in this context.

6) Assertion (A): If n(A)=m, then the number of reflexive relation on A is m.

Reason(R):A relation R on set A is said to be reflexive if $(a,a) \in R \ \forall a \in A$.

Solution: In a reflexive relation, every element of a set is connected to itself only. So number of reflexive relation is 2^{m^2-m}. **Answer: D**

7) Let A and B be two finite sets such that n(A)=5 and n(B)=2.Then

Assertion (A):Number of one-one functions from A to B is $5P_2$

Reason (R): Number of onto functions from A to B is 30.

Solution: Since n(A)> $n(B)$, Number of one – one functions from A to B is zero.

Assertion(A) is false. Reason (R) is true. **Answer: D**

9) Assertion (A) : If A and B are two sets having 3 and 5 elements respectively, then the total number of functions that can be defined from A to B is 5^3.

Reason (R): A function from set A to set B relates elements of set A to elements of set B.

Solution : A function from set A to set B relates every element of set A to a unique element in set B.

Consequently R is not true.

Since each element of set A can be associated to any one of five elements in B and there are 3 elements in set A

Total number of functions from A to B $= 5 \times 5 \times 5 = 5^3$. **Answer : C**

10) Assertion (A):The relation $f: \{1,2,3,4\} \to \{x,y,z,p\}$ defined by $f = \{(1,x),(2,y),(3,z)\}$ is a bijective function

Reason (R): The relation $f: \{1,2,3,4\} \to \{x,y,z,p\}$ defined by $f = \{(1,x),(2,y),(3,z),(4,p)\}$ is one-one.

Solution: Assertion is false since f is not a function .4 has no image under f. **Answer: D**

VERY SHORT ANSWER TYPE QUESTIONS

1) Let $A = \{a,b,c\}$ and the relation R be defined on A as follows: R = {(a, a), (b, c), (a, b)}. Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

Solution : We have relation $R = \{(a,a),(b,c),(a,b)\}$

To make R reflexive, we must add (b, b) and (c, c).
To make R transitive, we must add (a, c) to R.
Minimum number of ordered pairs to be added in R are (b, b), (c, c) and (a, c).

2) Let C be the set of complex numbers. Prove that the mapping $f: C \to R$ given by $f(z) = |z|, \forall z \in C$, is neither one-one nor onto.
Solution: We have $f: C \to R$ given by $f(z) = |z|, \forall z \in C$
$f(3+4i) = |3+4i| = \sqrt{3^2+4^2} = 5$
$f(3-4i) = |3-4i| = \sqrt{3^2+4^2} = 5$
Thus f(z) is many-one.
Also $|z| \geq 0, \forall z \in C$
But co-domain given is R.
Hence $f(z)$ is not onto.

3) Let A={1,2,3}, B={4,5,6,7} and let f={(1,4), (2,5), (3,6)} be a function from A to B. State whether f is one-one or not. Justify your answer.
Solution: Given f={(1,4), (2,5), (3,6)}
Since every element of A has one and only one image in B under f, f is a one-one function.

4) A function $f: A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If A={1,2,3,4} then find the set B.
Solution: When $x = 1, f(1) = 2.1 = 2$
When $x = 2, f(2) = 2.2 = 4$
When $x = 3, f(3) = 2.3 = 6$
When $x = 4, f(4) = 2.4 = 8$
Since f is both one-one and onto Range of f = Codomain of f
B={2,4,6,8}

5) State whether the following statement is true or false. Justify your answer.
"The sine function is bijective in nature when the domain is $[0, 4\pi]$".
Solution : Given function f(x)=sin x, x∈ $[0, 4\pi]$
Let $x_1, x_2 \in [0, 4\pi]$ such that $f(x_1) = f(x_2)$
⇒ $\sin x_1 = \sin x_2$
⇒ $x_1 = n\pi + (-1)^n x_2, n \in \{0,1,2,3\}$
⇒ $x_1 = x_2$ or $x_1 = \pi - x_2$ or $x_1 = 2\pi + x_2$ or $x_1 = 3\pi - x_2$
⇒ sin x is not one-one in $[0, 4\pi]$.
⇒ sine function is not bijective in $[0, 4\pi]$.
⇒ The given statement is false.

6) Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.
Solution: Let A ={1, 2, 3, 4, 5, 6}
Relation on R is defined as R = {(a, b) : b = a + 1}
In roster form
R={(1,2), (2,3), (3,4), (4,5), (5,6), (6,7)}
R is not reflexive since (1,1),(2,2),(3,3),(4,4),(5,5),(6,6) ∉R.
$(1,2) \in R$ but $(2,1) \notin R$.
R is not symmetric.
$(1,2), (2,3) \in R$ but $(1,3) \notin R$.
R is not transitive.
Hence R is neither reflexive, symmetric nor transitive.

7) Let A= {1,2,3,4}. Let R be the equivalence relation on A x A defined by

$(a,b)R(c,d)$ iff $a+d=b+c$. Find the equivalence class $[(1,3)]$.

Solution: Given on set A = {1,2,3,4} an equivalence relation R on A x A is defined by

$(a,b)R(c,d)$ iff $a+d=b+c$

Let (1,3) R (x, y) for all (x, y) ∈ A x A

$$\Rightarrow 1+y = 3+x$$

$$\Rightarrow y-x = 2$$

Therefore (x, y) will be (1,3) and (2,4)

Hence [(1,3)] = {(1,3), (2,4)}

8) A={1,3,5,7,....} and B={2,4,6,8,...}.Define a function from A to B that is neither one-one nor onto.

Solution: Given sets A={1,3,5,7,....} and B={2,4,6,8,...}.

Let f :A→ B be $defined$ by $f(x)=2, \forall x \in A$.

Since all the elements of A have the same image 2 in B, f is not one-one.

Only the element 2 in B have a pre-image in A. Hence f is not onto.

f is neither one-one nor onto.

9) Let n be a fixed positive integer. Define a relation R in Z as follows: ∀ a,b ∈ Z, a R b if and only if a – b is divisible by n . Show that R is an equivalence relation.

Solution: Given that ∀ a,b ∈ Z, a R b if and only if a – b is divisible by n .

aRa => (a-a) is divisible by n, which is true for any integer a, as zero is divisible by n.

Hence R is reflexive.

aRb => (a-b) is divisible by n

⇨ - (b-a) is divisible by n

⇨ (b-a) is divisible by n

⇨ bRa

⇨ R is symmetric.

Let aRb and bRc.

=> (a-b) is divisible by n and (b-c) is divisible by n

=> (a-b) + (b-c) is divisible by n

=>a-c is divisible by n.

=> aRc

Hence R is transitive.

Since R is reflexive, symmetric and transitive, R is an equivalence relation.

10) $f(x) = \frac{2\,tanx}{1-tan^2x}$. Find the range of f(x) for x ∈ R.

Solution: Given $f(x) = \frac{2\,tanx}{1-tan^2x}$

⇨ $f(x) = tan2x$

We know that $tan\,x \in (-\infty,\infty)$

$$tan2x \in (-\infty,\infty)$$

Range of f(x) is $(-\infty,\infty)$. i.e., R.

SHORT ANSWER TYPE QUESTIONS

1) Let f: A→ B be defined by $f(x) = \frac{x-2}{x-3}$, where A=R- {3} and $B = R - \{1\}$. Discuss the bijectivity of the function.

Solution: To check f is one-one: $f(x_1) = f(x_2)$

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

=> $(x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

=> $x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

=> $-3x_1 + 2x_1 = -3x_2 + 2x_2$

$\Rightarrow \quad -x_1 = -x_2$

$$\Rightarrow \quad x_1 = x_2$$

Hence if $f(x_1) = f(x_2)$ *then* $x_1 = x_2$

f is one-one

To check f is onto.

$$f(x) = \frac{x-2}{x-3}$$

Let f(x)=y such that y $\in B = R - \{1\}$

$$y = \frac{x-2}{x-3}$$

$\Rightarrow \quad y(x-3) = x-2$

$\Rightarrow \quad xy - 3y = x-2$

$\Rightarrow \quad xy - x = 3y-2$

$\Rightarrow \quad x(y-1) = 3y-2$

$\Rightarrow \quad x = \dfrac{3y-2}{y-1}$

For $y = 1, x$ *is not defined.*

But it is given that $y \in R - \{1\}$

Hence x = $\frac{3y-2}{y-1} \in R - \{1\}$

Checking value for y= f(x)

Putting value of x in f(x)

$$f(x) = f\left(\frac{3y-2}{y-1}\right)$$

$\Rightarrow \quad$ f(x) = $\dfrac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3}$

$\Rightarrow f(x) = \dfrac{3y-2-2(y-1)}{3y-2-3(y-1)}$

$\Rightarrow \quad$ f(x) = y

Thus for every $y \in B$, *there exists* $x \in A$ *such that* $f(x) = y$.

Hence f is onto.

Since f is one-one and onto f is a bijective function.

2) Let N be the set of natural numbers and R be the relation on NXN *defined by* $(a,b)\ R\ (c,d)$ *iff* $ad = bc$ *for all* $a,b,c,d \in N$. *Show that R is an equivalence relation.*

Solution: Given a relation R on N× N *defined by* $(a,b)\ R\ (c,d)$

iff $ad = bc$ *for all* $a,b,c,d \in N$.

Reflexive: Let (a,b) $\in N \times N$ *such that* $(a,b)\ R\ (a,b)$

⇨ ab=ba (Product of two natural numbers is commutative)

⇨ R is reflexive

Symmetric: Let (a,b), (c,d) $\in N \times N$ *such that* $(a,b)\ R\ (c,d)$

⇨ ad=bc => bc = ad => cb = da =>(c,d) R (a,b)

⇨ R is symmetric

Transitive: Let (a,b), (c,d),(e,f) $\in N \times N$ *such that*

$(a,b)\ R\ (c,d)$=> ad=bc -----(i) and

(c,d) R (e,f) => cf = de ------(ii)

Multiplying (i) and (ii) we get

acdf = bcde => af = be => (a,b) R (e,f)

R is transitive.

Since R is reflexive, symmetric and transitive, the given relation R on N x N, is an equivalence relation.

3) Check whether the relation S in the set of real numbers R defined by $S = \{(a,b): a - b + \sqrt{2} \text{ is an irrational number}\}$ is reflexive, symmetric or transitive.

Solution: Reflexive : For $a \in S$, $a - a + \sqrt{2} = \sqrt{2}$ *is an irrational number.*

⇨ $(a,a) \in S$

Thus S is a reflexive relation.

Symmetric : Let $(a,b) \in S$

⇨ $a - b + \sqrt{2}$ *is an irrational number*

But $b - a + \sqrt{2}$ *may not be an irrational number.*

For example: $(\sqrt{2}, 1) \in S \Rightarrow \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1$ *is an irrational number.*

$(1, \sqrt{2}) \notin S$ *as* $1 - \sqrt{2} + \sqrt{2} = 1$ *is not an irrational number.*

Here $(a, b) \in S$ but $(b,a) \notin S$. So S is not a symmetric relation.

Transitive : Let $(a, b) \in S$ => $a - b + \sqrt{2}$ *is an irrational number* and

$(b, c) \in S$ => $b - c + \sqrt{2}$ is an irrational number.

But $a - c + \sqrt{2}$ may not be an irrational number.

For Example :

$(1, \sqrt{3}) \in S$ *as* $1 - \sqrt{3} + \sqrt{2}$ *is an irrational number.*

$(\sqrt{3}, \sqrt{2}) \in S$ *as* $\sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$ *is an irrational number*

But $(1, \sqrt{2}) \notin S$ *as* $1 - \sqrt{2} + \sqrt{2} = 1$ *is not an irrational number.*

=> $(a, c) \notin S$. *So S is not a transitive relation.*

Thus S is reflexive but neither symmetric nor transitive relation.

4) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41, x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

Solution: We have $R = \{(x, y): x \in N, y \in N, 2x + y = 41\}$

Since $y \in N$, Domain = {1,2,3,....,20}

$R = \{(1,39), (2,37), (3,35), \ldots\ldots, (19,3), (20,1)\}$

Range = {1,3,5,.....,39}

R is not reflexive as $(2,2) \notin R$ as $2 \times 2 + 2 \neq 41$.

Also R is not symmetric as $(1,39) \in R$ *but* $(39,1) \notin R$.

Further R is not transitive as $(11,19) \in R, (19,3) \in R$ *but* $(11,3) \notin R$.

Hence R is neither reflexive, symmetric nor transitive.

Therefore R is not an equivalence relation.

5) A function $f: R \to R$ is defined by $f(x) = ax + b$, such that f(1)=1 and f(2)=3. Find function f(x). Hence check whether function f(x) is one-one and onto or not.

Solution: Given f(1)=1

⇨ $a + b = 1$------(1)

$f(2) = 3$

⇨ $2a + b = 3$-----(2)

Solving equations (1) and (2) we get a=2 and b = -1.

$f(x) = 2x - 1$ is the required function.

One-one : Let $x_1, x_2 \in R$ *such that* $x_1 \neq x_2$

=> $2x_1 \neq 2x_2$

=> $2x_1 - 1 \neq 2x_2 - 1$

Hence f(x) is a one-one function.

Onto: Let $y=f(x) \in R$

$$\Rightarrow \quad y=2x-1$$

$$\Rightarrow \quad x=\frac{y+1}{2}$$

Thus for all $y \in R, \exists\, x = \frac{y+1}{2} \in R$ such that $f(x) = f\left(\frac{y+1}{2}\right) = 2\left(\frac{y+1}{2}\right) - 1 = y$

f(x) is an onto function.

Hence f is one-one and onto.

6) Show that the function $f: R \to R$ *defined by* $f(x) = \frac{x}{x^2+1}, \forall x \in R$ *is neither one – one nor onto.*

Solution :

Let $x_1, x_2 \in R$ *such that* $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow \quad x_1(x_2^2 + 1) = x_2(x_1^2 + 1)$$

$$\Rightarrow x_1x_2^2 + x_1 = x_2x_1^2 + x_2$$

$$\Rightarrow x_1x_2^2 - x_2x_1^2 = x_2 - x_1$$

$\Rightarrow x_1x_2(x_2 - x_1) = x_2 - x_1$

$\Rightarrow x_1x_2(x_2 - x_1) - (x_2 - x_1) = 0$

$$\Rightarrow (x_2 - x_1)(x_1x_2 - 1) = 0$$

$$\Rightarrow x_2 - x_1 = 0 \quad or \quad (x_1x_2 - 1) = 0$$

$$\Rightarrow x_2 = x_1 \text{ or } x_1x_2 = 1$$

Here $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2 \quad or \quad x_1 = \frac{1}{x_2}$

f is not one-one.

Let $k \in R$ be any arbitrary element and let $f(x)=k$

$$\frac{x}{x^2+1} = k$$

$\Rightarrow \quad x = k(x^2 + 1)$

$\Rightarrow \quad kx^2 + k = x$

$$kx^2 - x + k = 0$$

⇨ $x = \frac{1 \pm \sqrt{1-4k^2}}{2k} \notin R, if\ 1 - 4k^2 < 0$

⇨ $or\ if\ (1 - 2k)(1 + 2k) < 0$

⇨ i.e., $k > \frac{1}{2}$ $or\ k < \frac{-1}{2}$

⇨ f is not onto.

Hence f is neither one-one nor onto.

7) Let $f: R \to R$ be the function defined by $f(x) = \frac{1}{2-\cos x}$ $\forall x \in R$. Then, find the range of f.

Solution:

$f: R \to R,\ f(x) = \frac{1}{2-\cos x}\ \forall x \in R$

Let $y = \frac{1}{2-\cos x}$

$$2y - y\cos x = 1$$

$\cos x = \frac{2y-1}{y}$

$\cos x = 2 - \frac{1}{y}$

We know that $-1 \leq \cos x \leq 1$

$$-1 \le 2 - \frac{1}{y} \le 1$$

$$-1-2 \le 2 - \frac{1}{y} - 2 \le 1-2$$

$$-3 \le -\frac{1}{y} \le -1$$

$$1 \le \frac{1}{y} \le 3$$

$$\frac{1}{3} \le y \le 1$$

Range is $\left[\frac{1}{3}, 1\right]$

8) Show that the function $f : (-\infty, 0) \to (-1,0)$ *defined by* $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ *is one – one and onto.*

Solution: Given $f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$

Since $x \in (-\infty, 0), |x| = -x$

Therefore $f(x) = \frac{x}{1-x}$

One-one: Let $x_1, x_2 \in (-\infty, 0)$

Now $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$

$\Rightarrow x_1(1 - x_2) = x_2(1 - x_1)$

$\Rightarrow x_1 - x_1x_2 = x_2 - x_2x_1$

$\Rightarrow x_1 = x_2$

=>Hence f is one-one.

Onto :

Let $y = \frac{x}{1+|x|}$

$y = \frac{x}{1-x}$ *Since* $x \in (-\infty, 0)$

$\Rightarrow y(1 - x) = x$

⇨ $y - yx = x$

⇨ $y = x + yx$

⇨ $Y = x(1 + y)$

⇨ $x = \frac{y}{1+y}$

For each $y \in (-1,0)$ *there exists* $x \in (-\infty, 0)$ *such that*

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\frac{y}{1+y}}{1-\frac{y}{1+y}} = y$$

Hence f is onto.

Thus f is both one-one and onto.

LONG ANSWER TYPE QUESTIONS

1) Prove that the relation R in the set of integers Z defined as R={(a, b): 2 *divides* (a + b)} *is an equivalence relation. Also determine* [3]

Solution: $a + a = 2a$, which is divisible by 2, ∀ a ∈ Z

$\Rightarrow (a,a) \in R, \forall a \in Z$

⇒R is reflexive.

Symmetric: Let (a, b) ∈ R ⇒ 2 divides (a+b)

⇒ 2 divides (b+a)

⇒ (b, a) ∈ R
R is symmetric
Transitive: Let (a, b), (b, c)∈R
⇒ 2 divides (a+b) and (b+c) both
⇒ (a + b) =2m and (b +c) = 2n
⇒a+2b+c=2m+2n
⇒a+c=2(m+n-b)
⇒2 divides (a+c) ⇒ R is transitive.
Since R is reflexive, symmetric and transitive,
R is an equivalence relation.

$$[3] = \{x: x \text{ is an odd integer}\}$$

Or $[3] = \{.....-1,1,3,5,7,.....\}$

2) If N denotes the set of all natural numbers and R is the relation on N × N defined by (a, b) R (c, d) if $ad(b + c) = b\,c(a + d)$. Show that R is an equivalence relation.

Solution:

Reflexive: By commutative law under addition and multiplication of natural numbers $b+a = a + b$ and $ab = ba\ \forall a, b \in N$.

$ab(b + a) = ba(a + b)\quad \forall a, b \in N$

=> $(a, b)R(a, b)$

Hence, R is reflexive.

Symmetric: Let $(a, b), (c, d) \in N \times N$ such that $(a, b)R\ (c, d)$

=> $ad(b + c) = bc(a + d)$.

=> $bc(a + d) = ad(b + c)$

=> $cb(d + a) = da(c + b)$

=> $(c, d)\ R\ (a, b)$

Hence, R is symmetric.

Transitive: Let (a, b), (c, d), (e, f) ∈ N X N such that
(a, b) R (c, d) and (c, d) R (e, f)
⇒ ad(b+c) = bc(a + d) and cf(d+e) = de(c +f)

=> $\frac{b+c}{bc} = \frac{a+d}{ad}$ and $\frac{d+e}{de} = \frac{c+f}{cf}$

=> $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ .... (1) and

$\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$ ....(2)

Adding (1) *and* (2)

=> $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

=> $\frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$

=> $\frac{e+b}{be} = \frac{f+a}{af}$

=>af(b+e)=be(a+f)
=>(a,b) R (e, f)
Hence, R is transitive.
Thus, R is reflexive, symmetric and transitive.
Therefore, R is an equivalence relation.

3) In the set of natural numbers N, define a relation R as follows:

$\forall n, m \in N$, n R m if on division by 5 each of the integers n and m leaves the remainder less than 5, i.e. one of the numbers 0, 1, 2, 3 and 4. Show that R is an equivalence relation. Also, obtain the pairwise disjoint subsets determined by R.

Solution:

R is reflexive since for each $a \in N$, aRa.

R is symmetric since if aRb, then bRa for $a, b \in N$.

Also, R is transitive since for $a, b, c \in N$, if aRb and bRc, then aRc.

Hence R is an equivalence relation in N which will partition the set N into the pairwise disjoint subsets.

The equivalent classes are as mentioned below:

$A_0 = \{5, 10, 15, 20 ...\}$

$A_1 = \{1, 6, 11, 16, 21 ...\}$

$A_2 = \{2, 7, 12, 17, 22, ...\}$

$A_3 = \{3, 8, 13, 18, 23, ...\}$

$A_4 = \{4, 9, 14, 19, 24, ...\}$

It is evident that the above five sets are pairwise disjoint and

$$A_0 \cup A_1 \cup A_2 \cup A_3 \cup A_4 = \bigcup_{i=0}^{4} A_i = N$$

4) Show that the function f: R→ R *defined as* $f(x) = x^2 + x + 1$ *is neither one – one nor onto. Also find all the values of* x *for which* $f(x) = 3$.

Solution:

Given a function f: R→ R *defined as* $f(x) = x^2 + x + 1$

Let $x_1, x_2 \in R$ *such that* $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1^2 - x_2^2) + (x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) + (x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow (x_1 - x_2) = 0$ *or* $(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ *or* $x_1 = -(x_2 + 1)$

$\Rightarrow$ f is not one-one.

Onto:

Let y=f(x)

$\Rightarrow Y = x^2 + x + 1$

$\Rightarrow x^2 + x + 1 - y = 0$

For $x \in R$, *discriminant* $D \geq 0$

$\Rightarrow 1^2 - 4 \times 1 \times (1 - y) \geq 0$

$\Rightarrow 1 - 4 + 4y \geq 0$

$\Rightarrow 4y - 3 \geq 0$

$\Rightarrow 4y \geq 3$

$\Rightarrow y \geq \frac{3}{4}$

$\Rightarrow y \in [\frac{3}{4}, \infty)$

$\Rightarrow$ Range of f(x) is $[\frac{3}{4}, \infty) \neq$ *Co – domain of f i.e., R*

$\Rightarrow$ f is not onto.

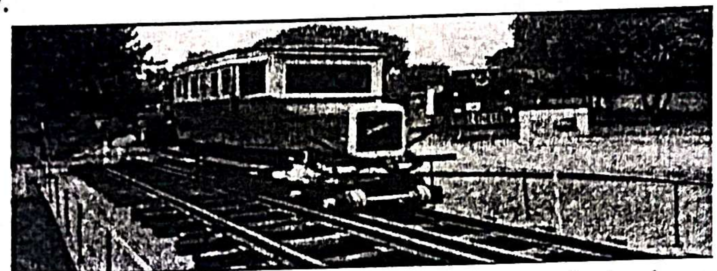
Hence f is neither one-one nor onto.

Also we have f(x)=3

$$\Rightarrow x^2 + x + 1 = 3$$
$$\Rightarrow x^2 + x - 2 = 0$$
$$\Rightarrow x^2 + 2x - x - 2 = 0$$
$$\Rightarrow x(x+2) - (x+2) = 0$$
$$\Rightarrow (x+2)(x-1) = 0$$
$$\Rightarrow x = 1, -2$$

CASE BASED QUESTIONS

1) Students of a school are taken to a railway museum to learn about railways heritage and its history.



An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by R = $\{(l_1, l_2): l_1 \text{ is parallel to } l_2\}$.
On the basis of the above information answer the following questions:
(i) Find whether the relation R is symmetric or not.
(ii) Find whether the relation R is transitive or not.
(iii) (a) If one of the rail lines on the railway track is represented by the equation y=3x+2, then find the set of rail lines in R related to it. OR
(b)Let S be the relation defined by S = $\{(l_1, l_2): l_1 \text{ is perpendicular to } l_2\}$.
Check whether the relation S is symmetric and transitive.

Solution:
(i)Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R \Rightarrow$ R is a symmetric relation.
(ii) Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2$ and Let $(l_2, l_3) \in R \Rightarrow l_2 \parallel l_3$
Since $l_1 \parallel l_2$ and $l_2 \parallel l_3, l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R$
Hence R is a transitive relation.
(iii)(a) The set is $\{l : l \text{ is a line of type } y = 3x + c, c \in R\}$
(b) Let $(l_1, l_2) \in S \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in S$
$\Rightarrow$ S is symmetric.
Let $(l_1, l_2) \in S \Rightarrow l_1 \perp l_2$ and
Let $(l_2, l_3) \in S \Rightarrow l_2 \perp l_3$
$$l_1 \perp l_2 \text{ and } l_2 \perp l_3 \Rightarrow l_1 \parallel l_3$$
$\Rightarrow (l_1, l_3)$ *is not an element of S.*
S is not a transitive relation.

2) A class room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A={1,2,3}.
$$R_1 = \{(2,3), (3,2)\}$$
$$R_2 = \{(1,2), (1,3), (3,2)\}$$
$$R_3 = \{(1,2), (2,1), (1,1)\}$$
$$R_4 = \{(1,1), (1,2), (3,3), (2,2)\}$$
$$R_5 = \{(1,1), (1,2), (3,3), (2,2), (2,1), (2,3), (3,2)\}$$
The students are asked to answer the following questions about the above relations:
(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

(iii) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iv) What pairs should be added to the relation R_2 to make it an equivalence relation ?

Solution:

(i) R_4 (ii) R_5 (iii) R_1 (iv){(1,1),(2,2),(3,3),(2,1),(3,1),(2,3)}

3) Let A be the set of 50 students of class XII in a school. Let $f : A \rightarrow N$, N is the set of natural numbers such that the function f(x) = Roll Number of student x. On the basis of the given information, answer the following:

(i) Is f a bijective function ?

(ii) Give reasons to support your answer to (i)

(iii) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where $R = \{(x, y): x, y \; are \; roll \; numbers \; of \; students \; such \; that \; y = 3x\}$. *List the elements of R.* Is the relation R reflexive, symmetric and transitive? Justify your answer.

Solution :

(i) *No*

(ii) No two different students of the class can have same roll number. Therefore, f must be one-one.

We can assume without any loss of generality that roll numbers of students are from 1 to 50. This implies that 51 in N is not roll number of any student of the class, so that 51 cannot be image of any element of X under f.

Hence, f is not onto.

(iii) $R =$ {(1,3),(2,6),(3,9),(4,12),(5,15),(6,18),(7,21),(8,24),(9,27),(10,30),(11,33),(12,36),(13,39),(14,42),(15,45),(16,48)}

R is not reflexive since (1,1),(2,2).... $\notin$R

R is not symmetric

Example: (1,3)$\in$R but (3,1) $\notin$R

R is not transitive.

Example: (1,3)$\in$R,(3,9)$\in$R but (1,9) $\notin$R

Since R is not reflexive, symmetric and transitive, R is not an equivalence relation.

CHAPTER-2: INVERSE TRIGONOMETRIC FUNCTIONS

Gist/Summary of the lesson:

In mathematics, the trigonometric functions are also called as circular functions, angle functions.

Inverse trigonometric functions are the inverse functions of the basic trigonometric functions which are sine, cosine, tangent, cotangent, secant, and cosecant functions.

Definition: Inverse trigonometric functions are the inverse functions of the basic trigonometric functions which are sine, cosine, tangent, cotangent, secant, and cosecant functions

Principal Value Branches:

FUNCTION	DOMAIN	RANGE (Principal Value Branch)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\operatorname{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	R	$(0, \pi)$

Formulae:

- $\sin^{-1}(-x) = -\sin^{-1} x, x \in [-1,1]$
- $\tan^{-1}(-x) = -\tan^{-1} x, x \in R$
- $cosec^{-1}(-x), = -cosec^{-1}(x),\ x \in R - (-1,1)$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x, x \in [-1,1]$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x, x \in R - (-1,1)$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$
- $\sin^{-1} x + \cos^{-1} x = \pi/2, x \in [-1,1]$
- $\tan^{-1} x + \cot^{-1} x = \pi/2, x \in R$
- $\sec^{-1} x + cosec^{-1} x = \pi/2, x \in R - [-1,1]$

Additional Formulae

- $\sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$
- $\sin^{-1} x - \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$
- $\cos^{-1} x + \cos^{-1} y = \cos^{-1}(xy - \sqrt{1-y^2}\sqrt{1-x^2})$
- $\cos^{-1} x - \cos^{-1} y = \cos^{-1}(xy + \sqrt{1-y^2}\sqrt{1-x^2})$
- $\tan^{-1} x + \tan^{-1} y = tan^{-1}\left(\frac{x+y}{1-xy}\right), if\ xy < 1$
- $\tan^{-1} x - \tan^{-1} y = tan^{-1}\left(\frac{x-y}{1+xy}\right), if\ xy > -1$
- $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$
- $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$
- $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$
- $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$
- $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$
- $3\tan^{-1} x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

MULTIPLE CHOICE QUESTIONS

1. The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$.

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{-\pi}{3}$ (d) $\frac{-\pi}{6}$

Solution: We have $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

$$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Ans: (a)

2. The principal value of $\sin^{-1}\left(\sin\left(\frac{3\pi}{5}\right)\right)$.

(a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{-2\pi}{5}$ (d) $\frac{\pi}{5}$

Solution: We have $\sin^{-1}\sin\left(\frac{3\pi}{5}\right) = \sin^{-1}\sin\left(\pi - \frac{3\pi}{5}\right) = \sin^{-1}\sin\left(\frac{2\pi}{5}\right) = \frac{2\pi}{5}$

Ans. (b)

3. The value of: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{6}$ (c) $\frac{-\pi}{3}$ d) 0

Solution: We have $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$.

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right) = \frac{-\pi}{3}$$

Ans: (c)

4. The value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ is

(a) 0 (b) 1 (c) -1 (d) 2

Solution: We have $\sin^{-1}(-x) = -\sin^{-1}(x)$

$$\therefore \sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right] = \sin\left[\frac{\pi}{3} - \left(\frac{-\pi}{6}\right)\right] = \sin\left(\frac{\pi}{2}\right) = 1$$

Ans: (b)

5. The principal value of $\cos^{-1}\left(\cos\left(\frac{-7\pi}{3}\right)\right)$ is

(a) $\frac{7\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{-\pi}{3}$ (d) $\frac{-7\pi}{3}$

Solution: We have $\cos(-x) = \cos x$

$$\therefore \cos^{-1}\cos\left(\frac{-7\pi}{3}\right) = \cos^{-1}\cos\left(\frac{7\pi}{3}\right) = \cos^{-1}\cos\left(2\pi + \frac{\pi}{3}\right) = \cos^{-1}\cos\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$$ **Ans: (b)**

6. The value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{6}$

Solution: $\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3})$

$$= \pi - \frac{\pi}{6} = 5\frac{\pi}{6}$$

$$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$$

Ans: (b)

7. The value of x if $\tan^{-1}\sqrt{3} + \cot^{-1}x = \frac{\pi}{2}$

(a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\pi}{6}$

Solution: $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1}\sqrt{3} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ $\therefore x = \cot\frac{\pi}{6} = \sqrt{3}$

Ans: (a)

8. The value of x if $sec^{-1} 2 + cosec^{-1}x = \frac{\pi}{2}$

(a) $\sqrt{3}$ (b) 2 (c) $\frac{\sqrt{3}}{2}$ (d) -2

Solution:

$$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}2 = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \quad \therefore x = \operatorname{cosec}\frac{\pi}{6} = 2$$

Ans: (b)

9. If $sin^{-1} x = y$ then the principal value of y is:

(a) $0 \le y \le \pi$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ (c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 < y < \pi$

Ans: (b)

10. If $tan^{-1} x = y$ then the principal value of y is:

(a) $0 \le y \le \pi$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ (c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 < y < \pi$

Ans: (c)

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1. ASSERTION (A): Principal value of $cos^{-1} cos\left(\frac{7\pi}{6}\right)$ *is* $\frac{5\pi}{6}$

   REASON (R): Range of principal branch of $\cos^{-1} x$ is $[0, \pi]$ and $\cos^{-1}\cos x = x$ if $x \in [0, \pi]$

   Ans: (a)

2. ASSERTION (A): Principal value of $sin^{-1} sin\left(\frac{13\pi}{6}\right)$ *is* $\frac{\pi}{6}$

   REASON (R): $\sin^{-1}(-x) = -\sin^{-1}(x)$

   Ans:(b)

3. ASSERTION (A): Principal value of $\sin^{-1}(-1) = \frac{-\pi}{2}$

   REASON (R): $\sin^{-1}(-x) = -\sin^{-1}(x)$

   Ans: (a)

4. ASSERTION (A): Principal value of $sin^{-1} sin\left(\frac{3\pi}{5}\right) = \frac{3\pi}{5}$

   REASON (R): $\sin^{-1}\sin(x) = x, x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

   Ans: (d)

5. ASSERTION (A): The principal value of $\cos^{-1}\left(\cos\left(\frac{-\pi}{2}\right)\right)$ is $\frac{-\pi}{2}$

   REASON (R): Cosine function is an even function, therefore $cos(-x) = cosx$.

Ans: (d)

6. ASSERTION (A): The principal value of $\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$.

REASON (R): Range of $\cos^{-1}x$ is $[0, \pi]$

Ans: (b)

7. ASSERTION (A): The principal value of $\tan^{-1}\sin\left(\frac{-\pi}{2}\right) = \frac{-\pi}{2}$.

REASON (R): $\tan^{-1}(-x) = -\tan^{-1}(x)$

Ans: (d)

8. ASSERTION (A): The principal value of $\tan^{-1}\tan\left(\frac{-\pi}{4}\right) = \frac{-\pi}{4}$.

REASON (R): Range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\tan^{-1}(\tan x) = x$ if $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Ans: (a)

9. ASSERTION(A): One branch of $\cos^{-1}x$ other than the principal value branch is $[\pi, 2\pi]$

REASON (R): $\cos\left(\frac{-\pi}{2}\right) = -1$

Ans: (c)

10. ASSERTION (A): A branch of $\sin^{-1}x$ other than principal branch is $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

REASON (R): $\sin\left(\frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

Ans: (c)

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the value of $\sin^{-1}(\sin\left(\frac{2\pi}{3}\right)) + \cos^{-1}(\cos\left(\frac{2\pi}{3}\right))$.

Solution: We have $\sin^{-1}(\sin\left(\frac{2\pi}{3}\right)) = \sin^{-1}(\sin\left(\pi - \frac{\pi}{3}\right)) = \sin^{-1}(\sin\left(\frac{\pi}{3}\right)) = \frac{\pi}{3}$

Value of $\cos^{-1}(\cos\left(\frac{2\pi}{3}\right)) = \frac{2\pi}{3}$

$\therefore$ $\sin^{-1}(\sin\left(\frac{2\pi}{3}\right)) + \cos^{-1}(\cos\left(\frac{2\pi}{3}\right)) = \frac{\pi}{3} + \frac{2\pi}{3} = \pi$

2. Find the value of: $\tan^{-1}\left[2\cos(\sin^{-1}\left(\frac{1}{2}\right))\right]$

Solution: $\tan^{-1}\left[2\cos(\sin^{-1}\left(\frac{1}{2}\right))\right] = \tan^{-1}\left[2\cos\left(\frac{\pi}{6}\right)\right] = \tan^{-1}\left[2 \times \frac{\sqrt{3}}{2}\right] = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

3. Find the value of: $\tan^{-1}\left[2\sin(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right))\right]$

Solution: $\tan^{-1}\left[2\sin(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right))\right] = \tan^{-1}\left[2\sin(2\frac{\pi}{6})\right] = \tan^{-1}\left[2 \times \frac{\sqrt{3}}{2}\right] = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

4. If $\cot^{-1}\left(\frac{1}{5}\right)$ then find the value of $\sin x$

Solution: $\cot x = \frac{1}{5}$ $\therefore$ $\sin x = \frac{5}{\sqrt{26}}$

5. Find the value of $\sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

Solution: $\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$,

$$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{6} + 2\times\frac{5\pi}{6} = \frac{3\pi}{2}$$

6. Show that for $|x| < 1, \sin\left(\tan^{-1} x\right) = \frac{x}{\sqrt{1+x^2}}$

Solution: Let $\tan^{-1} x = y$, $\tan y = x$

$\therefore$ $LHS = \sin y = \frac{x}{\sqrt{1+x^2}} = RHS$

7. Prove that : $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

Solution: Let $\sin^{-1}\frac{3}{4} = x$ $\therefore$ $\sin x = \frac{3}{4}$ $\therefore$ $\cos x = \frac{\sqrt{7}}{4}$

$$LHS = \tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}} = \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}} = \frac{4-\sqrt{7}}{3} = RHS$$

8. Find the value of $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$

Solution: Let $\tan^{-1}\frac{1}{5} = x$ $\therefore$ $\tan x = \frac{1}{5}$

$$\therefore \quad \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) = \tan\left(2x - \frac{\pi}{4}\right) = \frac{\tan 2x - \tan\frac{\pi}{4}}{1+\tan 2x \tan\frac{\pi}{4}} = \frac{-7}{17}$$

where, $\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2} = \frac{5}{12}$

9. Find the value of $\sin^{-1}\left(\frac{-1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}(1)$

Solution: $\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $\quad \tan^{-1}(1) == \frac{\pi}{4}$

$$\therefore \sin^{-1}\left(\frac{-1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}(1) = \frac{-\pi}{6} + \frac{2\pi}{3} + \frac{\pi}{4} = \frac{3\pi}{4}$$

10. Find the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$

Solution: Let $\sin^{-1}\left(\frac{3}{5}\right) = \theta$ $\therefore$ $\sin\theta = \frac{3}{5}$ $\therefore$ $\sin\left(2\sin^{-1}\frac{3}{5}\right) = \sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\times\frac{3}{5}\times\frac{4}{5} = \frac{24}{25}$$

SHORT ANSWER TYPE QUESTIONS

1. Write the principal value of $\cos^{-1}[\cos(680)^\circ]$

Solution: We know that, principal value branch of $\cos^{-1} x$ is $[0, 180°]$.

Since, $680° \notin [0, 180°]$, so write $680°$ as $2 \times 360° - 40°$

Now, $\cos^{-1}[\cos(680)°] = \cos^{-1}[\cos(2 \times 360° - 40°)]$

$= \cos^{-1}(\cos 40°)$ [$\because \cos(4\pi - \theta) = \cos\theta$]

Since, $40° \in [0, 180°]$

$\therefore \cos^{-1}[\cos(680°)] = 40°$ [$\because \cos^{-1}(\cos\theta) = \theta; \forall \theta \in [0, 180°]$]

2. Write the principal value of the following. $\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(\frac{-1}{2}\right)$

Solution: $\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(\frac{-1}{2}\right) = \cos^{-1}\frac{\sqrt{3}}{2} + \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) + \pi - \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{6} + \pi - \frac{\pi}{3} = \frac{5\pi}{6}$

3. Find the value of $\cos^{-1}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) + \operatorname{cosec}^{-1}\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$

Solution: $\cos^{-1}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) + \operatorname{cosec}^{-1}\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = \cos^{-1}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) + \sin^{-1}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) = \frac{\pi}{2}$

4. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$

Solution: Let $\tan^{-1}\frac{1}{5} = \theta \Rightarrow \tan\theta = \frac{1}{5}$

$\tan\left(2\tan^{-1}\frac{1}{5}\right) = \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{\frac{2\times1}{5}}{1-\left(\frac{1}{5}\right)^2} = \frac{5}{12}$

5. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x), x \in [\frac{1}{2}, 1]$

Solution: Consider, RHS $= \cos^{-1}(4x^3 - 3x)$

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

RHS $= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$ [$\because \cos 3A = 4\cos^3 A - 3\cos A$] $= 3\cos^{-1} x =$ LHS

6. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x \in [-\frac{1}{2}, \frac{1}{2}]$

Solution: Consider, RHS $= \sin^{-1}(3x - 4x^3)$ ......(i)

Let $x = \sin\theta$, then $\theta = \sin^{-1} x$

from Eq. (i), we get RHS $= \sin^{-1}(3\sin\theta - 4\sin^3\theta) = \sin^{-1}(\sin 3\theta)$

[$\because \sin 3A = 3\sin A - 4\sin^3 A$] $= 3\theta = 3\sin^{-1} x$ [$\because \theta = \sin^{-1} x$] $=$ LHS

7. Write in simplest form: $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

Solution: $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right) = \frac{\pi}{4} - x$

8. Find the value of $\sec(\tan^{-1}(-\sqrt{3})) =$

Solution: $\sec(\tan^{-1}(-\sqrt{3})) = \sec(-\tan^{-1}(\sqrt{3})) = \sec(\tan^{-1}(\sqrt{3}))$

$= \sec\left(\tan^{-1}\left(\tan\frac{\pi}{3}\right)\right) = \sec\frac{\pi}{3} = 2$

9. If $\cos^{-1}\frac{1}{x} = \theta$, then write $\tan\theta$ in terms of x

Solution: $\cos\theta = \frac{1}{x} \Rightarrow \sec\theta = x \Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{x^2 - 1}$

10. Prove that $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right) = \frac{1}{2}\cos^{-1} x$

Solution: $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right) = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ Let $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$=\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)=\tan^{-1}\left[\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right]=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}=\frac{1}{2}\cos^{-1}x$

LONG ANSWER TYPE QUESTIONS

1. Prove that $\sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{8}{17}\right)=\cos^{-1}\left(\frac{84}{85}\right)$

Solution: Let $\sin^{-1}\left(\frac{3}{5}\right)=A$ and $\sin^{-1}\left(\frac{8}{17}\right)=B$

Thus, we can write $\sin A=\frac{3}{5}$ and $\sin B=\frac{8}{17}$

Now, find the value of cosA and cosB

$\cos A=\sqrt{[1-\sin^2 A]}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

Thus, the value of $\cos A=\frac{4}{5}$

$\cos B=\sqrt{[1-\sin^2 B]}=\sqrt{1-\left(\frac{8}{17}\right)^2}=\sqrt{1-\frac{64}{289}}=\sqrt{\frac{225}{289}}=\frac{15}{17}$

Thus, the value of $\cos B=\frac{15}{17}$

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

substitute the values

$\cos(A-B)=\left(\frac{4}{5}\right)\times\left(\frac{15}{17}\right)+\left(\frac{3}{5}\right)\times\left(\frac{8}{17}\right)$

$\cos(A-B)=\frac{60+24}{17\times5}$

$\cos(A-B)=\frac{84}{85}$

$(A-B)=\cos^{-1}\left(\frac{84}{85}\right)$

Substituting the values of A and B $\sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{8}{7}\right)=\cos^{-1}\left(\frac{84}{85}\right)$

2. i) Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$.

ii) If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$; $xy<1$, then write the value of $x+y+xy$

Solution: i). The value of $\cos^{-1}\left(\frac{1}{2}\right)$ is $\frac{\pi}{3}$.

The value of $\sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{\pi}{6}$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+2\left(\frac{\pi}{6}\right),=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

ii) $\frac{x+y}{1-xy}=\tan\frac{\pi}{4}=1\Rightarrow x+y=1-xy\Rightarrow x+y+xy=1$

3. Write the principal value of $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)$.

Solution: $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)=\frac{\pi}{4}+\pi-\cos^{-1}\left(\frac{1}{2}\right)$

$=\frac{\pi}{4}+\pi-\cos^{-1}\left(\cos\frac{\pi}{4}\right)=\frac{\pi}{4}+\pi-\frac{\pi}{4}=\frac{11\pi}{12}$

4. Write the value of $\tan^{-1}\left(2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right)$

Solution: $\tan^{-1}\left(2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right)=\tan^{-1}\left(2\sin\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)\right)$

$=\tan^{-1}\left(2\sin\left(2\times\frac{\pi}{6}\right)\right)=\tan^{-1}\left(2\sin\left(\frac{\pi}{3}\right)\right)=\tan^{-1}\left(2\times\frac{\sqrt{3}}{2}\right)$

$=\tan^{-1}(\sqrt{3})=\tan^{-1}\left(\tan\frac{\pi}{3}\right)=\frac{\pi}{3}$

5. If $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c=\pi$ prove that a + b + c = abc

Solution: $\tan^{-1}a=A,\tan^{-1}b=B,\tan^{-1}c=C$

⇨ a=tan A b=tan B c= tan C

⇨ $A+B+C=\pi \Rightarrow A+B=\pi-C$

$\Rightarrow \tan(A+B) = \tan(\pi - C) = -\tan C$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C \Rightarrow \tan A + \tan B = \tan A \tan B \tan C - \tan C$

$\Rightarrow \tan A + \tan B + \tan C = \tan A \tan B \tan C \Rightarrow a + b + c = abc.$

6. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then show that $xy + yz + zx = 1$

Solution: $\tan^{-1} x = A, \tan^{-1} y = B, \tan^{-1} z = C \Rightarrow x = \tan A, y = \tan B, z = \tan C$

$\Rightarrow A+B+C = \frac{\pi}{2} \Rightarrow \tan(A+B) = \tan\left(\frac{\pi}{2} - C\right) = \cot C = \frac{1}{\tan C}$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{1}{\tan C} \Rightarrow \tan A \tan C + \tan B \tan C = 1 - \tan A \tan B \tan C$

$\Rightarrow \tan A \tan C + \tan B \tan C + \tan A \tan C = 1$

7. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$

Solution: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}+\sqrt{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\cos\frac{x}{2}\sin\frac{x}{2}}}{\sqrt{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}-\sqrt{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\cos\frac{x}{2}\sin\frac{x}{2}}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right) = \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right) = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$$

8. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, \frac{-1}{\sqrt{2}} \le x \le 1$

Solution: Let $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{\theta}{2}}-\sqrt{2\sin^2\frac{\theta}{2}}}{\sqrt{2\cos^2\frac{\theta}{2}}+\sqrt{2\sin^2\frac{\theta}{2}}}\right) = \tan^{-1}\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$$

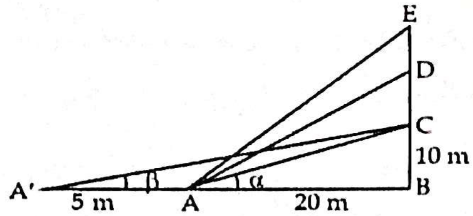
$$= \frac{\pi}{4} - \frac{\theta}{2} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

CASE BASED QUESTIONS

1) The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. 'A' is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby, Ram Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. 'C' is at the height of 10 metres from the ground level. For the viewer 'A', the angle of elevation of 'D' is double the angle of

elevation of ‘C’. The angle of elevation of ‘E’ is triple the angle of elevation of ‘C’ for the same viewer.

Look at the figure given and based on the above information answer the following:



(i) Measure of ∠CAB =

Ans. $\tan^{-1}(\frac{1}{2})$

(ii) Measure of ∠DAB =

Ans. $\tan^{-1}(\frac{4}{3})$

(iii) Measure of ∠EAB

Ans. $\tan^{-1}(\frac{11}{21})$

2). The following table gives inverse trigonometric functions along with domain and range

FUNCTION	DOMAIN	RANGE (Principal Value Branch)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	R	$(0, \pi)$

i. Value of $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right)$ is Ans: $\frac{11\pi}{12}$

ii. Value of $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\frac{1}{\sqrt{3}}$ is Ans. $\frac{\pi}{6}$

iii. Principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ Ans. $\frac{\pi}{3}$

OR

iv. Principal value of $\sec^{-1}(-2)$ Ans. $\frac{2\pi}{3}$